An EOQ model for deteriorating item with continuous linear time dependent demand with trade of credit and replenishment time being demand dependent

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Abstract: This study is about an inventory model with continuous linear time dependent demand rate with constant rate of deterioration in the consideration of partial backorder including delay in payment and time period is demand dependent. Demand is the fundamental attribute for consideration of inventory related problem. In reality, there is some inter connection of demand function among various time interval for which the demand cannot change drastically at some critical point during the appearance of another state of system. Thus, it is quite natural that the demand function should be continuous in nature in inventory management problem. So, here the most prominent part of our present study is the consideration of time dependent continuous demand in the proposed model. A supportive numerical example of the proposed model is illustrated for insightful investigation. The solution method and sensitivity analysis has also been presented.

Keywords: inventory; deterioration; delay in payment; trade credit period; backlog dependent.

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1 Introduction

Inventory problems become the most interesting topic to the researchers for the past few decades. The basic and most famous economic order quantity (EOQ) model was introduced by Harris (1913) and this model was widely discussed and implemented by Wilson (1934). Since this long century, several researchers contributed and implemented with various modifications on the traditional EOQ model in order to develop and improve such models to cope up with real life scenario in a much better way. In this context, the demand plays a prominent role in decisions related to inventory and production activities. In most of the classical inventory models demand rate is usually considered to be constant. Silver and Meal (1973) made significant effort on heuristic approach considering time varying demand rate. Donaldson (1977) introduced the classical deterministic EOQ model with no shortage where the demand was considered to be linear. The contribution of Bose et al. (1995) is worth mentioning in the same direction. In real life scenarios, many items such as chemicals, volatile liquids, blood banks, medicines and some other goods deteriorate during storage period which is non-negligible. In general, deterioration takes place due to the damage, decay, evaporation, spoilage and obsolescence of stored items. So, the management and holding of inventories of perishable items become an important problem for inventory managers. Covert and Philip (1973) developed an EOQ model for items with two variable Weibull distribution deterioration in constant demand interface. An inventory model with time dependent demand with constant deterioration and shortage model was proposed by Mishra and Singh (2010). Skouri (2018) extends the model studied by Bhunia and Maiti (1999) by considering generalised demand rate with time varying partial backlogging. Wu (2001) incorporated the deterioration inventory model with partial backlogging under ramp-type demand function, and afterwards Wu (2001) improved their model by considering generalised demand rate. Pervin et al. (2018) analysed the deterministic EOQ model with shortage under time dependent demand including partial backlogging to investigate the optimal retailers' replenishment decisions. Mashud et al. (2018) studied a stock and price dependent demand with deterioration under partially backlogged shortages. Taleizadeh and Nematollahi (2014) discussed an inventory control problem for deteriorating items with back ordering and financial considerations. Recently, many researcher, such as Aggrawal and Singh (2017), Singh et al. (2018) and Vandana and Sharma (2015), devoted their works to EOQ models with deterioration by considering several demands.

Chakraborty et al. (2018) discussed three variable weibul deteriorate model in two warehouse inventory problem. Chunhai et al. (2020) developed an deterministic inventory system with preservation technology. Mishra et al. (2019) introduce hybrid price stock dependent demand inventory model. Mahata et al. (2020) devolved an inventory model with anticipated stock-out situation. Ahmad and Benkherouf (2018) perishable products provides a constant rate with non-instantaneous deterioration, starting after a certain amount of time. Jana and Das (2021) consider a non-instantaneous deteriorating item EOQ model with stock dependent demand. Das et al. (2019) approach for the factorial order generalised inventory model with space constraints. Das and Roy (2015) incorporated fractional differential calculus method to the classical time dependent demand EOQ model. Das and Roy (2018) discussed a imprecise EOQ model under non-instantaneous deteriorating items in interval ecosystem. Gershwin et al. (2009) developed surplus production control with instantly taken up customer orders. Ghosh et al. (2011) attempted to solve a time dependent partial backlogging inventory problem with constant deterioration rate of deterioration. Ghosh and Chaudhuri (2015) proposed a multi-item inventory problem for deteriorating items with stock-sensitive demand under space constraint environment. Goswami and Chaudhuri (1991) presented an inventory model under a linearly time-varying demand function with exponentially deteriorating items, and shortages. Hill (1995) extended a linear increase in demand followed by a period of steady demand to a generalised model with a general power function. Jaggi et al. (2015) proposed an EOQ mathematical model to frame the optimal ordering policy for the buyer with allowable shortages and progressive credit periods. Kavitha and Senbagam (2018) proposed a mathematical inventory model with time dependent quadratic demand and linear storage cost function with two parameter Weibull deterioration rate. Khanra et al. (2011) developed an economic order quantity model for quadratic time-dependent demand for spare constant with trade off credit. Mandal and Pal (1998) investigated an inventory model with ramp type demand with shortage for perishable products. Manna and Chaudhuri (2006) studied inventory control model for time dependent deteriorating items with ramp type time-dependent demand and solved with two different environments considering once shortage and another without shortage. Patriarca et al. (2020) developed time dependent variable demand order level inventory model for deteriorating products which deal with the uncertainty of market in highly dynamic settings due to pandemic. Skouri et al. (2009) make important contribution in developing the ramp-type demand inventory model related with Weibul deterioration rate.

In practical business environments, delay in payments is usually considered to be an effective tool in order to insist the retailer to gather a higher quantity of items. Goyal (1985) was first researcher to develop the inventory model with permissible delay in payments. Then, Chang and Dye (2001) deals with partial backlogging EOQ model considering delay in payment for deteriorated items. Pal (2018) represented an economic lot size model considering delay in payment into account for sensitive quality demand. Recently, a fully backlogging EOQ model with trade credit policy for constant deteriorated items and shortage was discussed by Sen and Saha (2018). All of the above mentioned issues are separately considered in some inventory models. But, based on our knowledge, the work of the same kind of problems have been noticeably absent in the literature in the context where all the topics mentioned together. In this article, we address these issues by developing a comprehensive EOQ model precisely. In real life scenarios, it is hard to believe that there is no demand at the beginning, since there must be a positive demand level at the inception of such inventory. We address this issue by considering a continuous demand which is monotonic increasing before the stock out of inventory and which is monotonic decreasing after stock out period.

In this article, we consider a realistic continues demand function in which first demond increase with the time as stock is available and after the ending of stock the demond became decreasing in continues manner. Also we consider partial backlogging and shortage with trade of credit. This paper also acknowledge the time period of each cycle.

The rest of the paper is organised as follows. The assumption and notation of the proposed model is presented in Section 2. In Section 3, the formulation of mathematical model of the proposed model and its solution procedure is shown here. The numerical illustration is presented in Section 4. In Section 5, we represent the sensitivity analysis of the obtained result. Finally the conclusion is presented in Section 6.

2 Assumptions and notations

The mathematical model of the deterministic inventory replenishment problem is based on the following assumptions and notations:

1. Replenishment size is constant and replenishment rate is infinite.
2. Lead time is zero.
3. T is optimal duration of time of inventory cycle (per year).
4. S is the initial on-hand inventory level.
5. Q is the maximum inventory level for each ordering cycle.
6. C_h is the inventory holding cost (HC) per unit per unit time.
7. C_s is the shortage cost (SC) per unit per unit time.
8. C_d is the cost of each deteriorated unit.
9. C_l is the opportunity cost due to lose sales per unit.
10. δ permissible delay in trade of credit system.

11 $I(t)$ is the on-hand inventory at time t over $[0, T]$.

12 t_1 is the depletion time of inventory.

13 Shortages are allowed and partial back ordered. During the shortage period, the backlogging rate is variable and is dependent on the length of the wait time for the length replenishment. The longer the wait time is, the proportion of customers who would like to accept backlogging at time t is decreases with the wait time $(T - t)$ waiting for the next replenishment. To consider this situation the backlogging rate is defined to be when inventory is negative.

14 $\beta(y)$ is backorder function:

$$\beta(y) = \begin{cases} \beta_1 & \text{if } -B_1 < y \leq 0 \\ \beta_2 & \text{if } -B_2 < y \leq -B_1 \\ \beta_3 & \text{if } -B_3 < y \leq -B_2 \\ \dots \end{cases}$$

where $1 \geq \beta_1 \geq \beta_2 \geq \beta_3 \geq 0$; and $0 < B_1 < B_2 < B_3$.

15 $\theta(t)$ is the deterioration rate function.

16 c is the unit purchase cost per unit item.

17 p is the unit selling price per unit item.

18 A is the replenishment cost.

19 i_p is the interest paid per \$ per unit in stock per year by the retailer.

20 i_e is the interest earned (IE) per \$ per unit time per year by the retailer.

21 σ is permissible period (in year) of delay in settling the accounts with the supplier.

22 $TAC(t_1)$ is the total cost that consists of HC, deterioration cost, SC, lost sale (LS) cost, interest paying and earned cost.

23 $D(t)$ is the demand function, in this model we coniser continus demand function

3 Mathematical model formulation and solution

We developed the model in generalised form then the model is also shown effective in the particular method.

3.1 Generalised form

The proposed generalised inventory system for deteriorating item may be represented as follows:

Suppose $I(t)$ be the initial inventory level at any time t. S, i.e, $I(0)$ consider as an initial or starting inventory for the beginning of the period. Let us consider the deteriorating inventory model where demand is monotonic and continuous with time dependent. Then the inventory level comes to decrease from $t = 0$ to t_1, due to the continuously monotonic increasing demand rate throughout interval. The inventory level reaches zero at $t = t_1$, then the shortage occurs during the time interval $[t_1, T]$. Due to stock out the demand rate is monotonically decreasing along with the time t. In the shortage period $[t_1, T]$ the demand depends on partially time dependent backlogged rate. Delay in payment considerd in the proposed system in both the two cases before stock out and after stock out period. The characteristic of this proposed inventory system at any time t can be represented by the following diagram Figure 1. Based on the above description, during the time interval $[0, T]$.

The proposed instantaneous inventory level $I(t)$ at any time t during the cycle time T is governed by the following differential equation

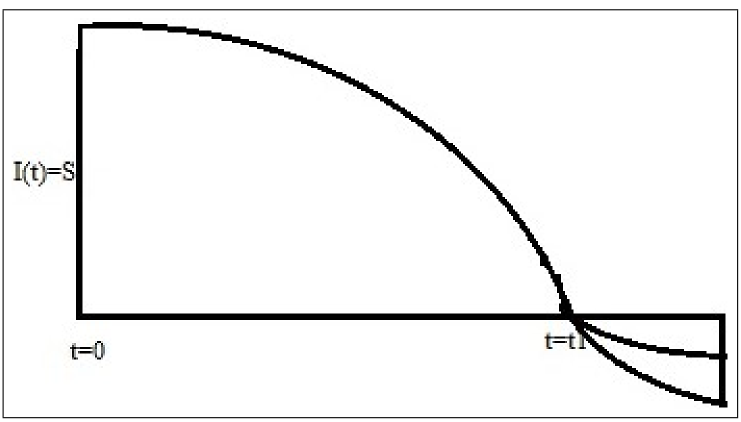
$$\frac{dI(t)}{dt} = \begin{cases} -\theta I(t) - D(t), & \text{if } 0 \le t \le t_1 \\ -\beta(t)D(t), & \text{if } t_1 \le t \le T. \end{cases} \tag{1}$$

where the boundary condition $I(0) = S$, $I(t_1) = 0$.

The generalised back order function $\beta(t)$ start acting after the end of stock in the time t within the time interval $[t_1, T]$

$$\beta(t) = \begin{cases} \beta_1 & \text{if } t_1 \le t < t_1 + \frac{B_1}{D(t)} \\ \beta_2 & \text{if } t_1 + \frac{B_1}{D(t)} \le t < t_1 + \frac{B_2}{D(t)} \\ \beta_3 & \text{if } t_1 + \frac{B_2}{D(t)} \le t \end{cases} \tag{2}$$

Figure 1 Graphical representation of inventory system



Solving the diff equation (1) and satisfying with the boundary condition, We have the Inventory at any time t in $[0, T]$ is

$$I(t) = \begin{cases} \exp(-\theta t) \int_t^{t_1} \exp(\theta t) D(t) dt & \text{if } 0 \le t < t_1 \\ -\int_{t_1}^{t} \beta(t) D(t) dt & \text{if } t_1 \le t < T \end{cases} \tag{3}$$

Putting $t = 0$ in the equation (3). The initial inventory of the system is

$$S = I(0) = \int_0^{t_1} D(t) \exp(\theta t) dt \text{ since } I(t_1) = 0 \tag{4}$$

The optimal order order quantity follows as within the time interval $[0, T]$

$$Q = \int_0^{t_1} D(t)\exp(\theta t)dt + \int_{t_1}^{T} \beta(t)D(t)dt \tag{5}$$

Cost of deterioration during time span $[0, T]$

$$\begin{aligned} DI &= c_d\left[S - \int_0^{t_1} D(t)dt\right] \\ &= c_d\left[\int_0^{t_1} D(t)\exp(\theta t)dt - \int_0^{t_1} D(t)dt\right] \end{aligned} \tag{6}$$

Cost of inventory holding during time span $[0, t_1]$

$$\begin{aligned} HC &= c_h\int_0^{t_1} I(t)dt \\ &= c_h\int_0^{t_1}\left(\exp(-\theta t)\int_t^{t_1}\exp(\theta t)D(t)dt\right)dt \end{aligned} \tag{7}$$

Shortage cost during time span $[0, T]$

$$\begin{aligned} SC &= c_s\int_{t_1}^{T} I(t)dt \\ &= -c_s\int_{t_1}^{T}\left(\int_{t_1}^{t}\beta(t)D(t)dt\right)dt \end{aligned}$$

LC cost during time duration $[t_1, T]$

$$LS = c_l(1-\beta(t))\int_{t_1}^{T} D(t)dt \tag{8}$$

Case 1 $(0 \leq \sigma \leq t_1)$

When permissible delay in payment σ is considered before the stock out period.
The interest is payable for the time period $[t_1 - \sigma]$.
The interest payable (IP) in the cycle $[\sigma, t_1]$

$$\begin{aligned} IP_1 &= i_p c\int_{\sigma}^{t_1} I(t)dt \\ &= i_p c\int_{\sigma}^{t_1}\left(\exp(-\theta t)\int_t^{t_1}\exp(\theta t)D(t)dt\right)dt \end{aligned} \tag{9}$$

The IE in the cycle $[0, t_1]$

$$IE_1 = i_e p\int_0^{\sigma} tD(t)dt \tag{10}$$

The total average cost of the inventory in the period $[0, T]$

$$
\begin{aligned}
TAC_1(t_1) &= \frac{1}{T}[Setup\ cost + Holding\ cost + Shortage\ cost \\
&+ Deterioration\ cost + Lost\ sale\ cost + Interest\ payable \\
&- Interest\ earning] \\
&= \frac{1}{T}\left[A + c_h \int_0^{t_1} \left(\exp(-\theta t) \int_t^{t_1} \exp(\theta t) D(t) dt \right) dt \right. \\
&+ c_s \int_{t_1}^{T} \left(\int_{t_1}^{t} \beta(t) D(t) dt \right) dt + c_l(1-\beta) \int_{t_1}^{T} D(t) dt \\
&+ c_d \left(\int_0^{t_1} D(t)\exp(\theta t) dt - \int_0^{t_1} D(t) dt \right) + i_e p \int_0^{\sigma} t D(t) dt \\
&\left. + i_p c \int_{\sigma}^{t_1} \left(\exp(-\theta t) \int_t^{t_1} \exp(\theta t) D(t) dt \right) dt \right] \qquad (11)
\end{aligned}
$$

Our aim is to minimise the total average cost of the inventory system $[0, T]$, we have to find t within the interval $[0, T]$, for that $\frac{d}{dt_1}\{TAC_1(t_1)\} = 0$ for some t_1 within the period $[0, T]$ and for that t_1, we have to show that $\frac{d^2TAC(t_1)}{dt_1^2} \geq 0$.

Diff the equation (11) where t within the period $[0, T]$. We have

$$
\begin{aligned}
\frac{d}{dt_1}\{TAC_1(t_1)\} &= \frac{1}{T}\left[\frac{c_h D(t_1)}{\theta}\{\exp(\theta t_1) - 1)\} - c_s(T - t_1)\beta(t_1)D(t_1) \right. \\
&\left. + c_d D(t_1)\{\exp(\theta t_1) - 1\} + i_p c D(t_1)\{\exp(\theta t_1) - 1\}\right] \qquad (12)
\end{aligned}
$$

The differential equation of the equation (12), where t within the period $[0, T]$.

We have

$$
\begin{aligned}
\frac{d^2}{dt_1^2}\{TAC_1(t_1)\} &= \frac{1}{T}\left[\frac{c_h}{\theta}\{D'(t_1)(\exp(\theta) - 1) - D(t_1)\exp(\theta t_1)\} \right. \\
&- c_s\Big\{(T - t_1)\{\beta'(t_1)D(t_1) + \beta(t_1)D'(t_1)\} - \beta(t_1)D(t_1)\Big\} \\
&\left. + (c_d + i_p c)\{D'(t_1)(\exp(\theta t_1) - 1) + D(t_1)\exp(\theta t_1)\}\right] \qquad (13)
\end{aligned}
$$

Total average cost will be minimum for the t_1 satisfying the equation

$$
\begin{aligned}
&\frac{c_h D(t_1)}{\theta}\{\exp(\theta t_1) - 1)\} - c_s(T - t_1)\beta(t_1)D(t_1) \\
&+ c_d D(t_1)\{\exp(\theta t_1) - 1\} + i_p c D(t_1)\{\exp(\theta t_1) - 1\} = 0 \qquad (14)
\end{aligned}
$$

and also satisfying the condition

$$
\begin{aligned}
&\frac{c_h}{\theta}\{D'(t_1)(exp(\theta) - 1) - D(t_1)exp(\theta t_1)\} \\
&- c_s\big\{(T - t_1)\{\beta'(t_1)D(t_1) + \beta(t_1)D'(t_1)\} - \beta(t_1)D(t_1)\big\} \\
&+ (c_d + i_p c)\{D'(t_1)(\exp(\theta t_1) - 1) + D(t_1)\exp(\theta t_1)\} \geq 0 \qquad (15)
\end{aligned}
$$

Case 2: $t_1 \le \sigma \le T$

When fully permissible delay in payment$\sigma$is considered after stock out period.

Then there will be no payment of interest, i.e., $IP_2 = 0$.

IE in the cycle $[0, t_1]$ is

$$IE_2 = i_e p\Big(\int_0^{\sigma} tD(t)dt + (\sigma - t_1)\int_0^{t_1} D(t)dt\Big) \tag{16}$$

In this case, total average cost of the inventory $[0, T]$

$$\begin{aligned}
TAC_2(t_1) &= \frac{1}{T}[Setup\ cost + Holding\ cost + Shortage\ cost \\
&\quad + Deterioration\ cost + Lost\ sale\ cost - Interest\ earning] \\
&= \frac{1}{T}\Bigg[A + c_h\int_0^{t_1}\left(\exp(-\theta t)\int_t^{t_1}\exp(\theta t)D(t)dt\right)dt \\
&\quad + c_s\int_{t_1}^{T}\left(\int_{t_1}^{t}\beta(t)D(t)dt\right)dt + c_l(1-\beta)\int_{t_1}^{T}D(t)dt \\
&\quad + c_d\left(\int_0^{t_1}D(t)\exp(\theta t)dt - \int_0^{t_1}D(t)dt\right) \\
&\quad + i_e p\left(\int_0^{\sigma}tD(t)dt + (\sigma - t_1)\int_0^{t_1}D(t)dt\right)\Bigg]
\end{aligned} \tag{17}$$

Diff the equation (17) with respect to t_1 within the period $[0, T]$. We have

$$\begin{aligned}
\frac{d}{dt_1}\{TAC_2(t_1)\} &= \frac{1}{T}\Bigg[\frac{c_h D(t_1)}{\theta}\{\exp(\theta t_1) - 1)\} - c_s(T - t_1)\beta(t_1)D(t_1) \\
&\quad + c_d D(t_1)\{\exp(\theta t_1) - 1\} + i_e P\{(\sigma - t_1)D'(t_1) \\
&\quad - \int_0^{t_1}D(t)dt\}\Bigg]
\end{aligned} \tag{18}$$

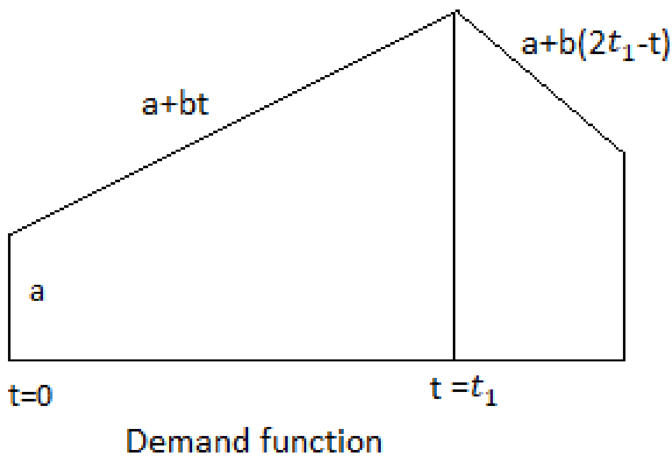
Diff the equation (18) with respect to t_1 within the period $[0, T]$. We have

$$\begin{aligned}
\frac{d^2}{dt_1^2}\{TAC(t_1)\} &= \frac{1}{T}\Bigg[\frac{c_h}{\theta}\{D'(t_1)(\exp(\theta) - 1) - D(t_1)\exp(\theta t_1)\} \\
&\quad - c_s\Big\{(T - t_1)\{\beta'(t_1)D(t_1) + \beta(t_1)D'(t_1)\} - \beta(t_1)D(t_1)\Big\} \\
&\quad + (c_d\{D'(t_1)(\exp(\theta t_1) - 1) + D(t_1)\exp(\theta t_1)\} \\
&\quad + i_e p\{(\sigma - t_1)D''(t_1) - 2D'(t_1)\}\Bigg]
\end{aligned} \tag{19}$$

3.2 Particular form

In this subsection, we developed a particular problem based on real life situations. In the particular case we consider the real life demand situation where demand rate is time dependent continuously monotonic increasing and this demand rate also allow to time dependent continuously monotonic decreasing after on stock as we find in real life, which is throughout continuous over the total period. Constant deterioration consider the this particular case. Back-order is time dependent. Fully permissible delay in payment consider in this problem.

Figure 2 Graphical representation of proposed demand function



The demand rate $D(t)$ is assumed to be continuous in nature and function of time:

$$D(t) = \begin{cases} a + bt & \text{if } 0 \leq t \leq t_1 \\ a + b(2t_1 - t) & \text{if } t_1 \leq t \leq T \end{cases} \tag{20}$$

$$\frac{dI(t)}{dt} = \begin{cases} -\theta I(t) - (a + bt), & \text{if } 0 \leq t \leq t_1 \\ -\beta_1\{a + b(2t_1 - t)\}, & \text{if } t_1 \leq t \leq T. \end{cases} \tag{21}$$

The solutions of the differential equation (21) with the boundary conditions $I(0) = S$, $I(t_1) = 0$.

$$I(t) = \frac{e^{\theta(t_1 - t)}}{\theta^2}\{(a + bt_1)\theta - b\} - \frac{1}{\theta^2}\{(a + bt)\theta - b\} \quad \text{if } \ 0 \leq t \leq t_1 \tag{22}$$

$$\begin{aligned} I(t) &= -\int_{t_1}^{t} \beta(t)\{a + b(2t_1 - t)\}dt, \quad \text{if } t_1 \leq T \leq t_1 + \frac{B_1}{D(t)} \\ &= -\int_{t_1}^{t} \beta_1\{a + b(2t_1 - t)\}dt \\ &= \beta_1\left\{a(t_1 - t) + \frac{1}{2}b(t_1^2 + t^2 - 2t_1 t)\right\} \end{aligned} \tag{23}$$

Initial inventory level is as follows

$$S = I(0) = \frac{e^{\theta t_1}}{\theta^2}\{(a + bt_1)\theta - b\} - \frac{1}{\theta^2}(a\theta - b) \tag{24}$$

The optimal order order quantity is as follows

$$\begin{aligned} Q &= \int_0^{t_1} D(t)e^{\theta t}dt + \int_{t_1}^{T} \beta(t)D(t)dt \\ &= \int_0^{t_1} (a + bt)e^{\theta t}dt + \int_{t_1}^{T} \beta_1\{a + (2t_1 - t)b\}dt \\ &= \frac{e^{\theta t_1}}{\theta^2}\{(a + bt_1)\theta - b\} - \frac{1}{\theta^2}(a\theta - b) + \beta_1\{a(T - t_1) \\ &+ b\left(2t_1T - \frac{3}{2}t_1^2 - \frac{1}{2}T^2\right)\Bigg\} \end{aligned} \tag{25}$$

Therefore, deterioration cost during the period $[0, t_1]$

$$\begin{aligned} DI &= c_d\left[S - \int_0^{t_1} D(t)dt\right] \\ &= c_d\left[\frac{e^{\theta t_1}}{\theta^2}\{(a + bt_1) - b\} - \frac{1}{\theta^2}(a\theta - b) - \left(at_1 + \frac{1}{2}bt_1^2\right)\right] \end{aligned} \tag{26}$$

The HC during the period $[0, t_1]$

$$\begin{aligned} HC &= c_h\int_0^{t_1} I(t)dt \\ &= c_h\int_0^{t_1} \left[\frac{e^{\theta(t_1 - t)}}{\theta^2}\{(a + bt_1)\theta - b\} - \frac{1}{\theta^2}\{(a + bt)\theta - b\}\right]dt \\ &= c_h\left[\frac{1}{\theta^3}(e^{\theta t_1} - 1)\{(a + bt_1)\theta - b\} - \frac{1}{\theta^2}\left\{\left(at_1 + \frac{1}{2}bt_1^2\right) - bt_1\right\}\right] \end{aligned} \tag{27}$$

The SC during the period $[t_1, T]$ is evaluated

$$\begin{aligned} SC &= c_s\int_{t_1}^{T} I(t)dt \\ &= c_s\beta_1\int_{t_1}^{T} \left\{a(t_1 - t) + \frac{1}{2}b(t_1^2 + t^2 - 2t_1t)\right\} \\ &= c_s\beta_1\left\{a\left(\frac{1}{2}T^2 + \frac{1}{2}t_1^2 - t_1T\right) + b\left(t_1T^2 - \frac{1}{6}T^3 - \frac{3}{2}t_1^2T + \frac{2}{3}t_1^3\right)\right\} \end{aligned} \tag{28}$$

Moreover, the amount of LSs during the period $[t_1, T]$ as follows

$$LS = c_l(1 - \beta_1)\int_{t_1}^{T} D(t)dt$$

$$= c_l(1-\beta_1)\int_{t_1}^{T}\{a+b(2t_1-t)\}dt$$

$$= c_l(1-\beta_1)\left\{a(T-t_1)+b\left(2t_1T-\frac{3}{2}{t_1}^2-\frac{1}{2}T^2\right)\right\} \tag{29}$$

Case 1: when $0 \leq \sigma \leq t_1$

Since the interest is payable during the time $[t_1-\sigma]$, the IP in the cycle $[\sigma, t_1]$ is

$$IP_1 = i_p c\int_{\sigma}^{t_1} I(t)dt$$

$$= i_p c\int_{\sigma}^{t_1}\left[\frac{e^{\theta(t_1-t)}}{\theta^2}\{((a+bt_1)\theta)-b\}-\frac{1}{\theta^2}\{(a+bt)\theta-b\}\right]dt$$

$$= i_p c\left[\frac{1}{\theta^3}\{e^{\theta(t_1-\sigma)}-1\}\{(a+bt_1)\theta-b\}\right.$$

$$\left.-\frac{1}{\theta^2}[\{a(t_1-\sigma)+\frac{1}{2}b(t_1^2-\sigma^2)\}\theta-b(t_1-\sigma)]\right] \tag{30}$$

IE in the cycle $[0, t_1]$ is

$$IE_1 = i_e p\int_0^{\sigma} tD(t)dt$$

$$= i_e p\int_0^{\sigma} t(a+bt)dt$$

$$= i_e p\int_0^{\sigma}(at+bt^2)dt$$

$$= i_e p\left(\frac{1}{2}a\sigma^2+\frac{b}{3}\sigma^3\right) \tag{31}$$

Case 2: when $t_1 \leq \sigma \leq T$

In this case the customer earns interest on the sales revenue up to the permissible delay period and no interest is payable during the period for the item kept in stock. Therefore

$$IP_2 = 0 \tag{32}$$

$$IE_2 = i_e p\left[\int_0^{t_1} tD(t)dt+(\sigma-t_1)\int_0^{t_1} D(t)dt\right]$$

$$= i_e p\left[\int_0^{t_1} t(a+bt)dt+(\sigma-t_1)\int_0^{t_1}(a+bt)dt\right]$$

$$= i_e p\left\{\left(\frac{a}{2}t_1^2+\frac{b}{3}t_1^3\right)+(\sigma-t_1)\left(at_1+\frac{b}{2}t_1^2\right)\right\} \tag{33}$$

Therefore the total average cost per unit time during the time interval $[0, T]$ is as follows:

Case 1 $(0 \le \sigma \le t_1)$.

$$\begin{aligned}
TAC_1(t_1) &= \frac{1}{T}[Setup\ cost + Holding\ cost + Shortage\ cost \\
&+ Deterioration\ cost + Lost\ sale\ cost + Interest\ payable \\
&- Interest\ earning] \\
&= \frac{1}{T}\left[A + c_h\left[\frac{1}{\theta^3}(e^{\theta t_1} - 1)\{(a+bt_1)\theta - b\}\right.\right. \\
&- \left.\frac{1}{\theta^2}\left\{\left(at_1 + \frac{1}{2}bt_1^2\right) - bt_1\right\}\right] + c_d\left[\frac{e^{\theta t_1}}{\theta^2}\{(a+bt_1) - b\}\right. \\
&- \left.\frac{1}{\theta^2}(a\theta - b) - \left(at_1 + \frac{1}{2}bt_1^2\right)\right] \\
&+ c_s\beta_1\left\{a\left(\frac{1}{2}T^2 + \frac{1}{2}t_1^2 - t_1T\right) + b\left(t_1T^2 - \frac{1}{6}T^3 - \frac{3}{2}t_1^2T\right.\right. \\
&+ \left.\left.\frac{2}{3}t_1^3\right)\right\} + c_l(1-\beta_1)\left\{a(T-t_1) + b\left(2t_1T - \frac{3}{2}{t_1}^2\right.\right. \\
&- \left.\left.\frac{1}{2}T^2\right)\right\} + i_pc\left[\frac{1}{\theta^3}\{e^{\theta(t_1-\sigma)} - 1\}\{(a+bt_1)\theta - b\}\right. \\
&- \left.\frac{1}{\theta^2}\left\{\left\{a(t_1-\sigma) + \frac{1}{2}b(t_1^2 - \sigma^2)\right\}\theta - b(t_1 - \sigma)\right\}\right] \\
&- \left.i_ep\left(\frac{1}{2}a\sigma^2 + \frac{b}{3}\sigma^3\right)\right]
\end{aligned} \tag{34}$$

Our aim is to find minimum variable cost per unit time. The necessary and sufficient conditions to minimise $TAC_{1.1}(t_1)$ for a given value t_1 are respectively $\frac{d(t_1)}{dT}\{TAC_1\} = 0$ and $\frac{d^2TAC_{1.1}(t_1)}{dT} \ge 0$ gives the following nonlinear equation in T.

Diff the equation (35) with respect to t_1 with the period $[0, T]$. We have

$$\begin{aligned}
\frac{d}{dt}\{TAC_1\} &= \frac{1}{T}\left[\frac{c_h}{\theta^2}[e^{\theta t_1}\{(a+bt_1)\theta - 2b\} - \{(a+bt_1) - 2b\}]\right. \\
&+ c_d\left[\frac{e^{\theta t_1}}{\theta}\left\{(a+bt_1) - b + \frac{b}{\theta}\right\} - (a+bt_1)\right] \\
&+ c_s\beta_1\{a(t_1 - T) + b(T^2 - 3t_1T + 2t_1^2)\} \\
&+ c_l(1-\beta_1)\{b(2T - 3t_1) - a\} \\
&+ \left.\frac{i_pc}{\theta}(a+bt_1)\{e^{\theta(t_1-\sigma)} - 1\}\right]
\end{aligned} \tag{35}$$

Diffrentiial equation of the equation (36) with respect to t_1 with the period $[0, T]$. We have

$$\frac{d^2}{dt_1{}^2}\{TAC_1\} = \frac{1}{T}\left[c_h\left[\frac{1}{\theta^2}e^{\theta t_1}\{(a+bt_1)\theta - 3b\} - \frac{b}{\theta^2}\right]\right.$$
$$+ c_d\left[e^{\theta t_1}\left\{(a+bt_1) + \frac{b}{\theta}\right\} - b\right]$$
$$+ c_s\beta_1\{a + b(4t_1 - T)\} - 3c_l b(1-\beta_1)$$
$$\left. + \frac{i_p c}{\theta}[e^{\theta(t_1-\sigma)}\{(a+bt_1)\theta + b\} - b]\right] \quad (36)$$

Case 2 When fully permissible delay in paymentσis considered after stock out period, i.e., $t_1 \le \sigma \le T$.

Then there will be no payment of interest, i.e., $IP_2 = 0$,

$$TAC_2(t_1) = \frac{1}{T}\left[A + c_h\left[\frac{1}{\theta^3}(e^{\theta t_1} - 1)\{(a+bt_1)\theta - b\}\right.\right.$$
$$\left. - \frac{1}{\theta^2}\left\{\left(at_1 + \frac{1}{2}bt_1^2\right) - bt_1\right\}\right] + c_d\left[\frac{e^{\theta t_1}}{\theta^2}\{(a+bt_1) - b\}\right.$$
$$\left. - \frac{1}{\theta^2}(a\theta - b) - \left(at_1 + \frac{1}{2}bt_1^2\right)\right]$$
$$+ c_s\beta_1\left\{a\left(\frac{1}{2}T^2 + \frac{1}{2}t_1^2 - t_1T\right) + b\left(t_1T^2 - \frac{T^3}{6} - \frac{3}{2}t_1^2T\right.\right.$$
$$\left.\left. + \frac{2}{3}t_1^3\right)\right\} + c_l(1-\beta_1)\left\{a(T-t_1) + b\left(2t_1T - \frac{3}{2}t_1{}^2\right.\right.$$
$$\left.\left.\left. - \frac{1}{2}T^2\right)\right\} - i_e p\left\{\left(\frac{a}{2}t_1^2 + \frac{b}{3}t_1^3\right) + (\sigma - t_1)\left(at_1 + \frac{b}{2}t_1^2\right)\right\}\right] \quad (37)$$

Diffrentiating the equation (37) with respect to t_1 with the period $[0, T]$. We have

$$\frac{dTAC_2(t_1)}{dt} = \frac{1}{T}\left[\frac{c_h}{\theta^2}[e^{\theta t_1}\{(a+bt_1)\theta - 2b\} - \{(a+bt_1) - 2b\}]\right.$$
$$+ c_d\left[\frac{e^{\theta t_1}}{\theta}\left\{(a+bt_1) - b + \frac{b}{\theta}\right\} - (a+bt_1)\right]$$
$$+ c_s\beta_1\{a(t_1 - T) + b(T^2 - 3t_1T + 2t_1^2)\}$$
$$+ c_l(1-\beta_1)\{b(2T - 3t_1) - a\}$$
$$\left. - i_e p\left\{\left(at_1 - \frac{1}{2}bt_1^2\right) + (\sigma - t_1)(a+bt_1)\right\}\right] \quad (38)$$

Diffrentiating the equation (38) with respect to t_1 with the period $[0, T]$. We have

$$\frac{d^2TAC_{1.2}(t_1)}{dt_1{}^2} = \frac{1}{T}\left[c_h\left[\frac{1}{\theta^2}e^{\theta t_1}\{(a+bt_1)\theta - 3b\} - \frac{b}{\theta^2}\right]\right.$$

$$+ c_d \left[e^{\theta t_1} \left\{ (a + bt_1) + \frac{b}{\theta} \right\} - b \right]$$
$$+ c_s \beta_1 \{a + b(4t_1 - T)\} - 3c_l b(1 - \beta_1)$$
$$+ i_e pb(3t_1 - \sigma) \Big] \tag{39}$$

4 Numerical illustration

The numerical examples given below covers all the two cases that arise in the model.

Example 1 [Case 1(1)]: Minimum average cost $TAC_1(t_1)$ of equation (34).

Figure 3 Average cost vs. holding cost (see online version for colours)

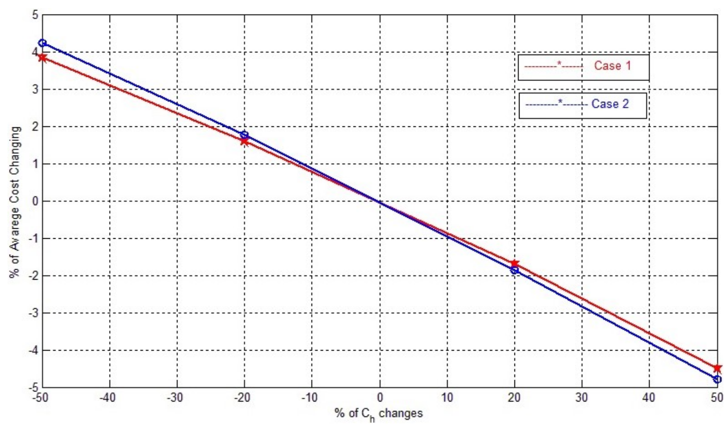
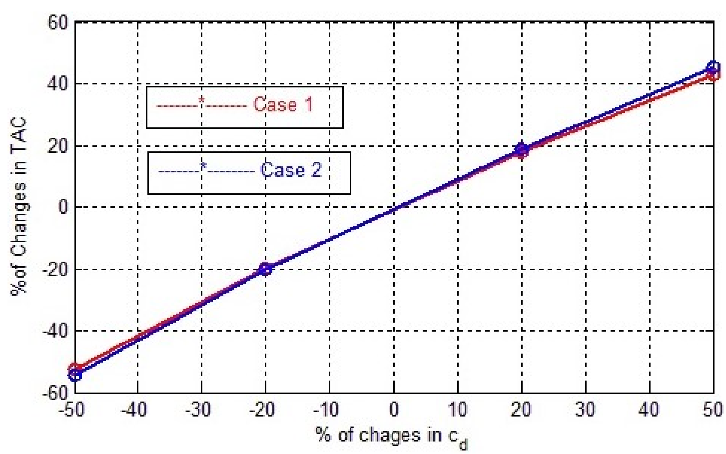


Figure 4 Average cost vs. deterioration cost (see online version for colours)



Let us take the parameter values of the inventory system as A = \$100 per order, a = 50 units per cycle, b = 1 units per cycle, C_h = \$0.2 per unit per cycle, C_s = \$2 per unit per cycle, C_d = \$1 per unit, C_l = \$3 per unit, c = \$2 per unit, p = \$5 per unit, I_e = 8% per year, I_p = 10% per year, θ = 0.4 year, σ = 0.3 and T = 1 year.

Solving equation (38), we have t_1^* = 0.6608 year and the minimum total average cost is $TAC_1(t_1^*)$ = \$268.127.

Example 2 (Case 2): Minimum average cost $TAC_2(t_1)$ of equation (38).

Figure 5 Total average cost vs. shortage cost (see online version for colours)

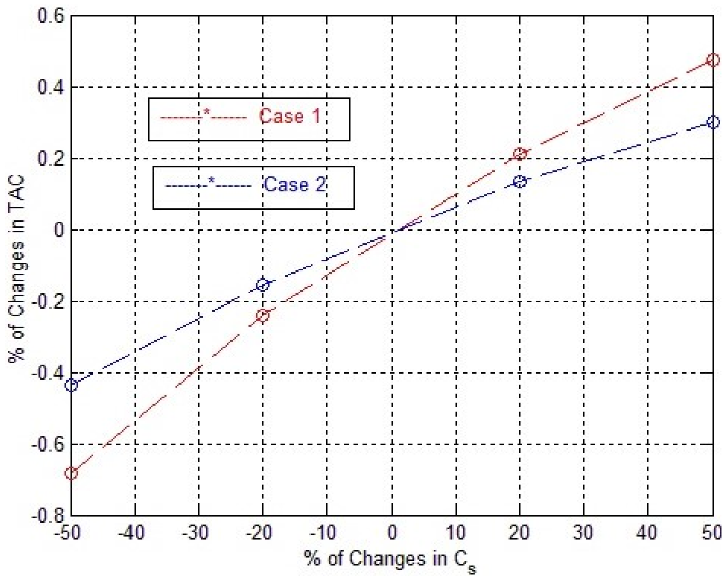


Figure 6 Total average cost vs. cost of lost sale (see online version for colours)

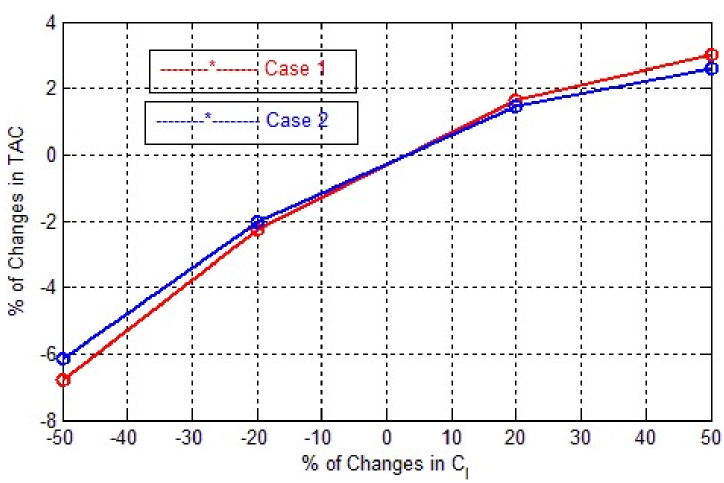


Figure 7 Total average cost vs. trade of credit (see online version for colours)

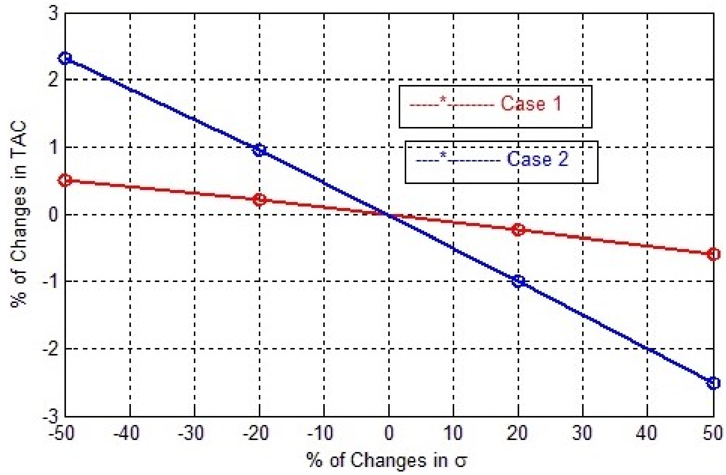


Figure 8 Total average cost vs. deterioration (see online version for colours)

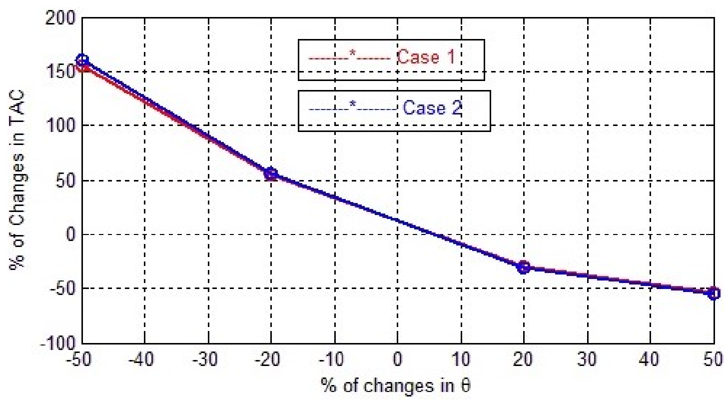
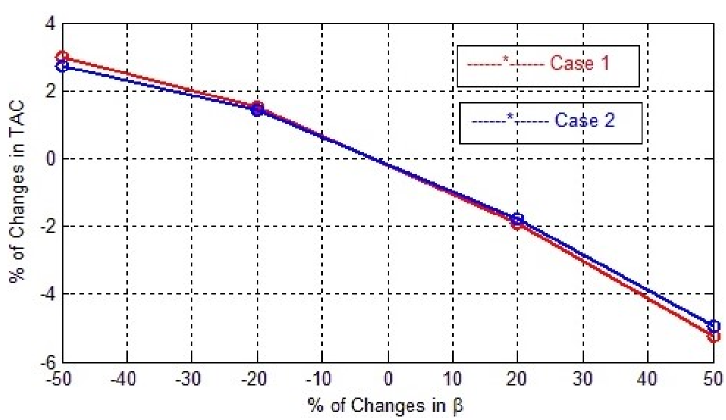


Figure 9 Total average cost vs. backlogging (see online version for colours)



Let us take the parameter values of the inventory system as A = \$100 per order, a = 50 units per cycle, b = 1 units per cycle, C_h = \$0.2 per unit per cycle, C_s = \$2 per unit per cycle, C_d = \$1 per unit, C_l = \$3 per unit, c = \$2 per unit, p = \$5 per unit, I_e = 8% per year, I_p = 10% per year, θ =0.4 year, σ = 0.9 and T = 1 year.

Solving equation (38), we have t_1^* = 0.8287 year and the minimum total average cost is $TAC_2(t_1^*)$ = \$261.5062.

5 Sensitivity analysis

The effect of the changes in the system parameters C_h, C_s, C_d, C_l, c, p, I_e, I_p, θ, σ, a, b, A and T on the optimal value t_1^*, optimal on-hand inventory (S^*), optimal order quantity (Q^*) and optimal total average cost per unit time (TAC^*) in the EOQ model of Case 1 are now studied. The sensitivity analysis is performed by changing each of the parameters by –50%, –20%, 20% and 50%, taking one parameter at a time and keeping the remaining parameters unchanged. The results are shown in Tables 1 and 2 respectively according to the Cases 1 and 2.

It has been seen from the Table 1 that the TAC^* is highly sensitive with respect to changes in values of C_h, C_d, C_l, a, θ and T. This is moderately sensitive respect to changes in values of C_h, C_l, α, β. Besides, value of TAC^* is mild sensitive with respect to the changes in parameters, such as C_s, c, b, i_p. Whereas both t_1^* and S^* are highly sensitive respect to changes in values of C_h, C_d, C_l, a, θ, T and β. Again t_1^* is moderately sensitive respect to changes in C_s, c, b and it is mild sensitive with respect to the change in i_p, σ, whereas S^* is moderately sensitive with respect to change in C_s, b, σ and it is mild sensitive with respect to the change in c, i_p. Q^* is highly sensitive with respect to change in C_h, C_d, C_l, a, θ, T and β whereas it is moderately sensitive with the change in C_s and mildly sensitive with the change in c, b, i_p and σ.

In Case 2, i.e., from Table 2 it has been observed that TAC^* is highly sensitive with respect to the change in C_d, a, θ, T and it is seen to be moderately sensitive with respect to the change in C_h, C_l, β. There is a light impact of TAC^* with respect to the change in parameter C_s, c, b, i_p, σ. Here we see that t_1^* is highly sensitive with respect to the change in parameters C_h, C_d. C_l, θ, T, β and it is moderately sensitive with the change in C_s, c and t_1^* is lightly sensitive with the change in parameters a, b, i_p, σ. Again S^* is here highly sensitive with respect to the change in parameters C_h, C_S, C_d, a, θ, T, β and it moderately sensitive with respect to the change in C_l, c. There is a mild impact on S^* with respect to the change in the parameters b, i_p, σ. Whereas Q^* is highly sensitive with respect to the change in parameters C_h, C_d, a, θ, T and it is seen to be moderately sensitive with the change in parameters C_s, C_l, β and there is a little impact on Q^* with respect to the change in parameters c, b, i_p, σ.

From Tables 1, 2 and Figure 3, we have observed that total average cost and holding cost behaving as moderate sensitive with negative correlation, we also observed that the parameter are more consistent in Case 1 then Case 2.

From Tables 1, 2 and Figure 4, we have observed that total average cost and deterioration cost behaving as highly sensitive with positive correlation, we observed that the parameter are showing almost same characteristics in the both cases, i.e., Case 1 then Case 2.

Table 1 Sensitivity analysis table for Case 1

Parameter	*% change*	*% change of* t_1	*% change of S*	*% change of Q*	*% change of TAC*
C_h	50	16.0562	18.8019	9.6470	–4.4622
	20	6.6434	7.6779	3.9131	–1.7164
	–20	–6.9612	–7.8949	–3.9841	1.6177
	–50	–18.0387	–20.1481	–10.0851	3.8435
C_s	50	7.8541	9.0927	4.6381	0.4747
	20	3.4806	4.0051	2.0366	0.2089
	–20	–4.0406	–4.6010	–2.3269	–0.2412
	–50	–11.5315	–12.9959	–6.5363	–0.6829
C_d	50	–55.2512	–58.6591	–28.5417	42.9980
	20	–24.2887	–26.8972	–13.4007	18.0714
	–20	28.1780	33.5615	17.3673	–19.5925
	–50	3.0886	3.1937	0.2714	–13.9791
C_l	50	39.9062	48.3219	25.2093	3.0127
	20	16.2228	19.0013	9.7504	1.6477
	–20	–16.5708	–18.5459	–9.2933	–2.2485
	–50	–42.1005	–45.5012	–22.3657	–6.7674
c	50	–2.0581	–2.3501	–1.1901	0.1200
	20	–0.8475	–0.9693	–0.4912	0.0491
	–20	1.3389	0.8777	0.5107	–0.0508
	–50	2.2397	2.5728	1.3072	–0.1301
a	50	1.8916	52.9011	51.2741	47.4963
	20	0.9534	21.1720	20.5159	19.0000
	–20	–1.4225	–21.1678	–20.5149	–19.0039
	–50	–5.7657	–52.9738	–51.3206	–47.5276
b	50	–2.8602	–2.9407	–1.2999	0.6195
	20	–1.1350	–1.1639	–0.5128	0.2506
	–20	1.1350	1.1613	0.5105	–0.2544
	–50	2.8299	2.8893	1.2671	–0.6428
i_p	50	–2.0581	–2.3501	–1.1901	0.1200
	20	–0.8475	–0.9693	–0.4912	0.0491
	–20	0.8777	1.0063	0.5107	–0.0508
	–50	2.2397	2.5728	1.3072	–0.1301
θ	50	14.8608	27.2602	16.9784	–53.7136
	20	6.6435	10.9249	6.5640	–29.6474
	–20	–4.7669	–7.8660	–4.7356	55.1141
	–50	27.6332	21.6776	7.8602	154.7949
σ	50	1.8765	2.1545	1.0943	–0.5946
	20	0.7718	0.8847	0.4489	–0.2278
	–20	–0.7718	–0.8829	–0.4474	0.2149
	–50	–1.9673	–2.2466	–1.1378	0.5139
T	50	25.0151	29.6623	42.3309	–20.7544
	20	10.1695	11.8110	16.8309	–10.6885
	–20	–10.3814	–11.7183	–27.5034	45.4393
	–50	–26.3771	–29.1263	–41.4079	68.4152

Table 1 Sensitivity analysis table for Case 1 (continued)

Parameter	*% change*	*% change of* t_1	*% change of S*	*% change of Q*	*% change of TAC*
β	50	–27.1338	–29.9307	–0.9230	–5.2512
	20	–11.8795	–13.3817	–2.2175	–1.8888
	–20	13.5291	15.7867	5.3741	1.5323
	–50	37.5303	45.2929	21.1040	2.9735

Table 2 Sensitivity analysis table for Case 2

Parameter	*% change*	*% change of* t_1	*% change of S*	*% change of Q*	*% change of TAC*
C_h	50	12.3205	14.9045	8.9069	–4.7965
	20	5.1164	6.1122	3.6311	–1.8556
	–20	–5.3819	–6.3130	–3.7183	1.7658
	–50	–13.9858	–16.1628	–9.4510	4.2340
C_s	50	3.2702	3.8940	2.3098	0.3019
	20	1.4481	1.7188	1.0179	0.1328
	–20	–1.6894	–1.9944	–1.1784	–0.1534
	–50	–4.8510	–5.6953	–3.3559	–0.4349
C_d	50	–49.4630	–53.8031	–30.5037	45.1829
	20	–21.9138	–24.9818	–14.5094	18.9294
	–20	25.7874	31.9431	19.2966	–20.3948
	–50	75.2625	101.8582	63.8968	–54.6610
C_l	50	33.1362	41.5831	25.2661	2.5884
	20	13.4789	16.3390	9.7734	1.4594
	–20	–13.7927	–15.9449	–9.3250	–2.0297
	–50	–35.0670	–39.0871	–22.4446	–6.1507
p	50	7.9401	9.5322	5.6762	–1.0033
	20	3.1978	3.8072	2.2583	–0.4289
	–20	–3.2340	–3.8075	–2.2466	0.4624
	–50	–8.1574	–9.5226	–5.5957	1.2138
a	50	1.5325	52.2807	51.1482	47.3814
	20	0.7602	22.0039	21.1059	19.0058
	–20	–1.1464	–20.9146	–20.4607	–18.9573
	–50	–4.6217	–52.3240	–51.1742	–47.4099
b	50	–2.3048	–2.3064	–1.2266	0.6888
	20	–0.9171	–0.9143	–0.4847	0.2783
	–20	0.9171	0.9106	0.4812	–0.2817
	–50	2.2927	2.2697	1.1962	–0.7104
i_e	50	7.9401	9.5322	5.6762	–1.0033
	20	3.1978	3.8072	2.2583	–0.4289
	–20	–4.0545	–4.7669	–2.8107	0.5827
	–50	–8.1574	–9.5226	–5.5957	1.2138
θ	50	10.4380	24.2709	18.2527	–55.3716
	20	4.7906	9.7092	7.0649	–30.5277
	–20	–3.3426	–7.1147	–5.2450	56.6123
	–50	24.7737	17.1322	6.0780	160.1857

Table 2 Sensitivity analysis table for Case 2 (continued)

Parameter	*% change*	*% change of* t_1	*% change of S*	*% change of Q*	*% change of TAC*
σ	50	8.1574	9.7966	5.8346	–2.5295
	20	3.2822	3.9085	2.3184	–0.9867
	–20	–3.3064	–3.8924	–2.2965	0.9530
	–50	–8.3142	–9.7030	–5.7008	2.3190
T	50	20.7916	25.5290	34.2004	–19.0477
	20	8.4590	10.1642	12.6246	–9.3462
	–20	–8.6521	–10.0914	–15.7218	16.9541
	–50	–21.9742	–25.0480	–36.6088	66.7459
β	50	–25.8115	–29.2291	–13.3162	–4.9400
	20	–11.3189	–13.1411	–5.2128	–1.7646
	–20	12.9118	15.6360	8.7414	1.4168
	–50	35.8272	45.1753	30.4881	2.7227

From Tables 1, 2 and Figure 5, we have observed that total average cost and shortage cost behaving as mild sensitive with positive correlation, we also observed that the parameter is seen to be more consistent in Case 2 than Case 1.

From Tables 1, 2 and Figure 6, we have observed that total average cost and cost of LS behaving as moderate sensitive with positive correlation, we also observed that the parameter are more consistent in Case 2 than Case 1.

From Tables 1, 2 and Figure 7, we have observed that total average cost is mild sensitive to trade of credit in Case 1 and moderate sensitive in Case 2 with negative correlation, we also observed that the parameter are much more consistent in Case 1 than Case 2.

From Tables 1, 2 and Figure 8, we have observed that total average cost and deterioration cost behaving as highly sensitive with negative correlation, we also observed that the parameter are showing almost same characteristics in both cases.

From Tables 1, 2 and Figure 3, we have observed that total average cost and backlogging behaving as moderate sensitive with negative correlation, we here observed that the parameter are more consistent in Case 2 than Case 1.

6 Conclusions

In this article, a generalised inventory model for deteriorating items with partial backlogging under permissible delay in payment and shortage have been developed. Hare demand in the proposed model is assumed to be continuous function of time which indeed monotonic throughout the entire replenishment interval. In this article, an inventory model for deteriorating items with partial backlogging under shortages have been developed. Here the demand in the proposed model is assumed to be continuous function of time which is indeed monotonic throughout the entire replenishment interval. The most prominent part of the demand function is that it is increasing with time but appeared to be decreasing during stock out period. Due to shortage of the product/item, the demand of the product decreases with time and that is very commonly occur in the market. So, in realistic perspective this kind of dual form of demand function have been considered in this proposed model. The backordered appeared in the stock out period which is assumed to be monotonic and time dependent which indeed shows real impact on a practical marketplace. The delay in payment has also been imparted to this proposed model. This present study suggested that the consideration of continuous demand is very significant which should not be ignored. So, for widespread vision of any sustainable business policy, the policy maker should not avoid such realistic situation occurred in the market. Based on these frameworks, an analytic formulation of this proposed model has been presented and the solution procedure has also been described to find its optimum total cost and the depletion time period. In addition, a numerical example has been presented to show the practical applicability of the proposed mode. Finally, the sensitivity of the solution has been discussed by changing the values of different key parameters. There so many complicacies are generally occurred in inventory model in its different aspect. So, based on the present study, the scope of future works is ample. There should be more investigations are required to observe the characteristic of the proposed model when lead time is taken into the consideration. Another approaches of this proposed model can be developed in various imprecise environment (like fuzzy, intuitionistic fuzzy, neutrosophic, etc.) to describe the uncertainty for the sake of more realistic sense.

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